T10/08-121r0
Limitations of df/dt Specification
for SSC Profiles

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Overview

- Previous Material
- Revised Simulation Methodology
- Limitations of Previous Proposals
- Value of Residual Jitter From SSC Slope
  - Derivation of Final Jitter Caused by a Frequency Ramp
  - Calculation of the Optimal Window for Slope Measurement
- JTF Residual Jitter vs SSC Profile df/dt (No HF Content)
- Frequency Response of the Slope Measurement over a Window
- Response of the Slope Measurement To High Frequency Jitter
- Improving the Slope Measurement With Low-Pass Filtering
- Conclusions
A proposal for JTF-based and df/dt-based specifications of the SSC profiles was developed in previous material:

- 08-027r3: “Toward SSC Modulation Specs and Link Budget”
- 08-032r4: “Proposed modifications to SSC profile definition “

This proposal was included into sas2r14.
- Created various SSC frequency modulation and jitter profiles
- SSC profiles are created directly for a 6Gb/s 1010 pattern
  - SSC jitter is not filtered through a PLL as in 08-027, which allows for the addition of high frequency jitter.
- Residual jitter is obtained by passing SSC jitter through JTF
Limitations of Previous Proposals

- 08-027r3 and 08-032r4 established a proportional relationship between the SSC slope and the residual jitter after the JTF.
- This presentation shows that the relationship between the SSC slope and the JTF filtered jitter holds only for low frequency content.
Final value of the residual jitter when the jitter produced by a frequency ramp is filtered by the JTF

\[
\lim_{t \to \infty} Jitter(t) = \lim_{t \to 0} \frac{2\pi}{s} \frac{\text{frequency deviation rate}}{s^2} \left( \frac{s^3 \cdot Tb + s^2}{s^3 \cdot Tb + s^2 + s \cdot K \cdot Ta + K} \right) \cdot \frac{1}{2\pi} = \frac{\text{frequency deviation rate}}{K}
\]

- Phase is integral of frequency
- Frequency ramp (triangular modulation)
- JTF
- Conversion from radians to ratio of the bit rate

For the clean SSC profiles used in this analysis, a very good match is obtained between the residual jitter predicted by a typical JTF without peaking and the residual jitter obtained using the frequency deviation rate calculated over a ~0.3 \(\mu\)s window (0.266\(\mu\)s is ideal window size)

- A maximum frequency deviation rate specification is a necessary but non-sufficient condition to guarantee link robustness
  - Averaging the slope over 0.3 \(\mu\)s window does not produce the proper high frequency response
The approximation of the JTF by a fixed averaging window is a first order approximation of the JTF transfer function.

\[
JTF(s) = \frac{\Theta_{\text{out}}(s)}{\Theta_{\text{in}}(s)} = \frac{s^3 \cdot T_b + s^2}{s^3 \cdot T_b + s^2 + s \cdot K \cdot T_a + K}
\]

The SSC profile is expressed in frequency, which is proportional to the derivative of the phase. Thus, we could re-write the JTF as a ratio between the output phase and the input frequency. Furthermore, we can include the conversion from radians to ratio of the bit rate to get the relative output jitter:

\[
JTF'(s) = \frac{\text{Jitter}_{\text{out}}(s)}{F_{\text{in}}(s)} = \frac{1}{2\pi} \cdot \frac{\Theta_{\text{out}}(s)}{F_{\text{in}}(s)} = \frac{1}{2\pi} \cdot \frac{2\pi}{s} \cdot \frac{\Theta_{\text{out}}(s)}{\Theta_{\text{in}}(s)} = \frac{(s^2 \cdot T_b + s)}{s^3 \cdot T_b + s^2 + s \cdot K \cdot T_a + K}
\]

This function can be expanded in a Taylor series:

\[
JTF'(s) = \frac{s}{K} + \frac{s^2 (T_b - T_a)}{K} + O(s^3)
\]
This has to be compared to using the average slope of the SSC profile over some time window. This operation can be seen as the frequency variation between the end and beginning of the window, divided by the window time.

\[
\text{Average SSC slope} = \frac{f(t) - f(t - \Delta T)}{\Delta T}
\]
This average slope process can then be evaluated in the frequency domain:

\[ a(t) = \frac{f(t) - f(t - \Delta T)}{\Delta T} \quad \rightarrow \quad A(s) = \frac{F(s) \cdot (1 - e^{-s \Delta T})}{\Delta T} \]

\[ \text{Slope}(s) = \frac{A(s)}{F(s)} = \frac{1 - e^{-s \Delta T}}{\Delta T} \]

To match the JTF, we are allowed to multiply this equation by a fixed constant, \( C \) (which should be equal to \( 1/K \), as what the first limit indicated for a constant slope)

\[ JTF_{\text{slope}}(s) = C \cdot \text{Slope}(s) \]

Again, we use a Taylor expansion

\[ JTF_{\text{slope}}(s) \approx C \left( s - s^2 \frac{\Delta T}{2} + O(s^3) \right) \]
Finally, we equate the two approximate expressions, ignoring the remaining high-order terms:

\[
JTF'_{\text{slope}}(s) = JTF'(s)
\]

\[
Cs - Cs^2 \frac{\Delta T}{2} = \frac{s}{K} + \frac{s^2(Tb-Ta)}{K}
\]

This results in the two solutions:

\[
C = \frac{1}{K}
\]

\[
\Delta T = 2 \cdot (Ta-Tb)
\]

As expected from the limit equation:

For a constant slope, we get Jitter = slope/K

Optimal slope window size to match the JTF knee (0.266us for nominal JTF)

Thus, the best approximation to match the JTF by the slope method is to compute the SSC profile slope over a window of time \(\Delta T = 2 \cdot (Ta-Tb)\) and to divide the result by K. This gives a result in a ratio to the bit rate (i.e. ppm-like). It will match up to the second derivative (i.e. terms in \(s^2\)).

Note that \(\Delta T\) is measured backwards from the current time (when \(Ta>Tb\)), to get the best curve fitting.
As presented in 08-027r3, in the absence of high frequency content, there is a very good match between Jitter calculated with the JTF and jitter calculated from the slope of the SSC profile.
As shown earlier, the jitter resulting from the average slope process in the frequency domain is:

\[ JTF_{\text{slope}}(s) = \frac{jitter_{\text{OUT}}(s)}{F(s)} = \frac{1}{K} \cdot \text{Slope}(s) = \frac{(1-e^{-s\Delta})}{K \cdot \Delta T} \]

To compare this response to the JTF frequency response, we need to express \( A(s) \) as a function of the input jitter:

\[ F(s) = s \cdot \Theta(s) = s \cdot 2\pi \cdot j\text{itter}_{\text{IN}}(s) = s \cdot j\text{itter}_{\text{IN}}(s) \]

\[ j\text{itter}_{\text{OUT}}(s) = s \cdot j\text{itter}_{\text{IN}}(s) \cdot (1-e^{-s\Delta}) \]

\[ K \cdot \Delta T \]

To compare this response to the JTF frequency response, we need to express \( A(s) \) as a function of the input jitter:
- $\Delta T = 0.266\text{us}$
- $\Delta T = 1.5 \text{us}$
The slope-based pseudo-JTF response matches well the real JTF up to \( f = \approx 1/\Delta T \) but:

- It amplifies high frequency jitter with a gain that increases 20dB per decade
- There are periodic nulls in the response above \( f = \approx 1/\Delta T \)

The slope-based pseudo-JTF is thus unlikely to match the real JTF for real-world SSC profiles with high frequency content.
• +/-300ppm random noise added to triangular SSC profile
Jitter from slope ($\Delta T=0.267\text{us}$) is higher than jitter from JTF
● Jitter from slope ($\Delta T = 0.267\text{us}$) is higher than jitter from JTF
  - and does not track it at all…
- Jitter from slope ($\Delta T=1.5\text{us}$) is lower than jitter from JTF and does not track any better
- $\Delta T = 0.266\text{us}$
- \( \Delta T = 1.5 \text{us} \)
Filtering gets rid of the high frequency jitter amplification.

There are still periodic nulls in the response above \( f \approx 1/\Delta T \).

The slope-based pseudo-JTF is thus unlikely to match the real JTF for real-world SSC profiles with high frequency content.
- Jitter from slope ($\Delta T = 0.267\text{us}$) is now lower than jitter from JTF
  - Effect of the HF nulls
• Jitter from slope ($\Delta T=0.267 \text{us}$) is now smaller than jitter from JTF
  - but still does not track very well…
Jitter from slope ($\Delta T=1.5\text{us}$) is much lower than jitter from JTF.
The slope-based pseudo-JTF response matches well the real JTF up to \( f = \approx 1/\Delta T \)
- With 1.5us window, it can cover \( \approx 20 \) harmonics of the SSC modulation
- With 0.27us window, it can cover \( \approx 100 \) harmonics of the SSC modulation

High-frequency jitter causes the slope-based pseudo-JTF to diverge from the real JTF
- In the presence of high frequency noise, the slope measurement of an SSC profile is not a good predictor of that profile’s compliance to the jitter specifications.

It is proposed to keep the JTF as the only filtering method for transmitter jitter measurements, with or without SSC.