

Effects of Periodic Structures on Transmission Lines

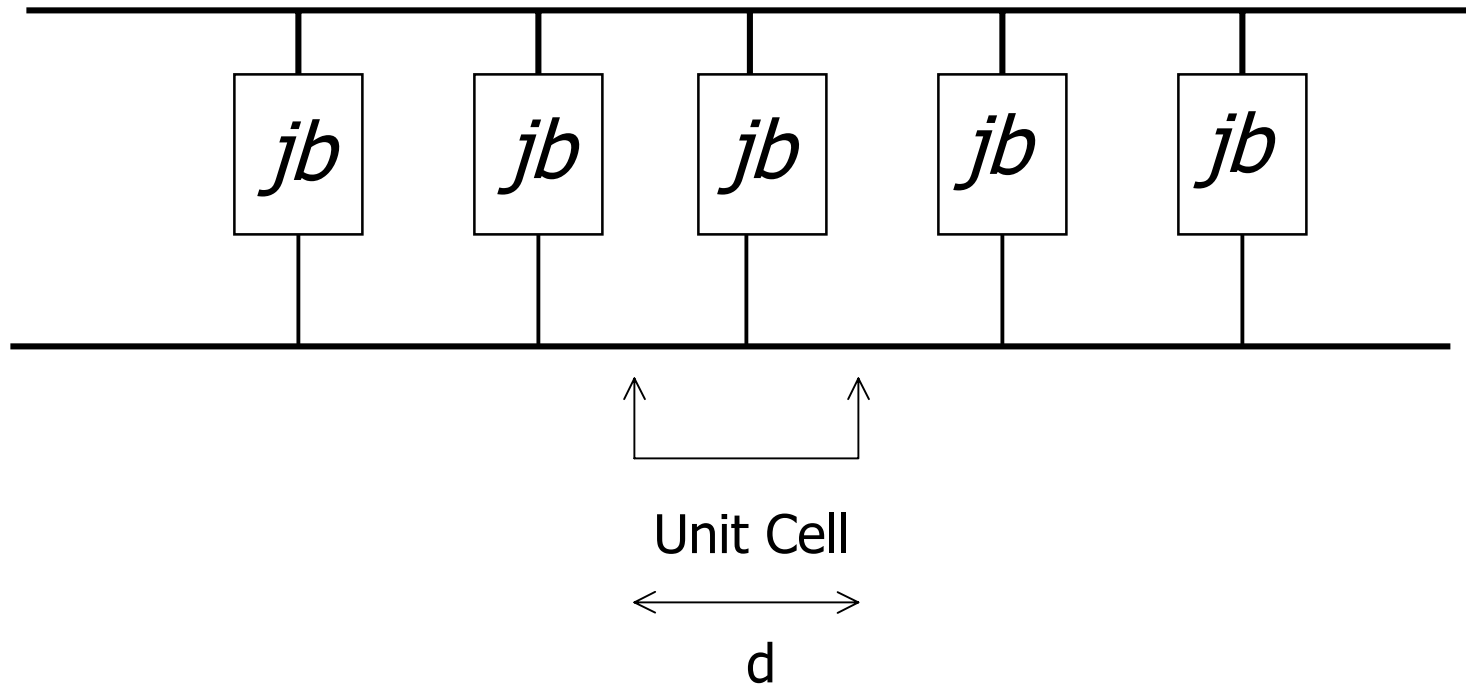
Why examine periodic structures?

- Periodic structures - a definition
 - Transmission lines loaded at periodic intervals with identical reactive elements
- The following are examples of periodic structures:
 - A backplane loaded with SCSI devices
 - A ribbon cable loaded with SCSI devices
 - A bare twisted - flat ribbon cable with or without connectors
 - Periodic stubs on a microstrip line.
- The theory of periodic structures:
 - Provides insight into the behavior of backplanes and cables
 - Explains this behavior quantitatively

Periodic structures

- Are characterized by a uniform distribution of reactive elements
- Have pass-band and stop-band properties
- Have slow wave properties
- Can exhibit significant impedance shifts
- Can be described mathematically through the use of voltage and current transfer functions

An equivalent circuit



Some definitions

- Propagation constant or wavenumber $k = \sqrt{\mu\varepsilon}$
- Complex propagation constant $\gamma = j\omega\sqrt{\mu\varepsilon}$
- Phase constant $\beta = \text{Im}(\gamma)$
- Attenuation constant $\alpha = \text{Re}(\gamma)$

Characteristics

- Unloaded line:
 - Has a propagation constant of k
 - Has a characteristic impedance of Z_0
- Structure consists of a number of unit cells
 - Consist of a length d of transmission line
 - Have a shunt susceptance b across the midpoint of the cell
 - The susceptance is normalized to Z_0
 - Can be represented by a cascade of identical 2-port networks

Network representation

- Structure is a cascade of identical 2 port networks
- Each cell is represented by an identical ABCD matrix
- The ABCD matrix represents
 - Section of transmission line of length $d/2$
 - A shunt susceptance of b
 - Another section of transmission line of length $d/2$
- Voltage and currents on either side of each cell are represented by

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}$$

ABCD Matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left(\cos \frac{\theta}{2} - \frac{b}{2} \sin \theta \right) & j \left(\sin \theta + \frac{b}{2} \cos \theta - \frac{b}{2} \right) \\ j \left(\sin \theta + \frac{b}{2} \cos \theta + \frac{b}{2} \right) & \left(\cos \theta - \frac{b}{2} \sin \theta \right) \end{bmatrix}$$

- $\theta = k \bullet d$ where k is the propagation constant of the unloaded line

Current and voltage phase

- The voltage and current phase only differ by the propagation factor $e^{-\gamma z}$

$$V_{n+1} = V_n e^{-\gamma z}$$

$$I_{n+1} = I_n e^{-\gamma z}$$

- Resulting in the following representation

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} V_{n+1} e^{\gamma d} \\ I_{n+1} e^{\gamma d} \end{bmatrix}$$

Solution of matrix

- The former equation reduces to the following

$$AD + e^{2\gamma d} - (A + D)e^{\gamma d} - BC = 0$$

- Since $\gamma = \alpha + j\beta$

- We have the following

$$\cosh \gamma d = \cosh \alpha d \cos \beta d + j \sinh \alpha d \sin \beta d = \cos \theta - \frac{b}{2} \sin \theta$$

- And since the right-hand side of the equation must be purely real, either
 - $\alpha = 0$, and $\beta \neq 0$
 - or
 - $\beta = 0$ or π , and $\alpha \neq 0$

Solution case $\alpha = 0$, and $\beta \neq 0$

- Corresponds to non-attenuating propagating wave
- Defines a passband
- Solved for β if magnitude of right-hand side is less than unity
- Infinite number of values of β

$$\cos \beta d = \cos \theta - \frac{b}{2} \sin \theta$$

Solution case $\alpha \neq 0, \beta = 0, \pi$

- Wave is attenuated
- Defines stopband of structure
- Causes reflections back to input
- Has only one solution for positively traveling waves
 - $\alpha > 0$ for positively traveling waves
 - $\alpha < 0$ for negatively traveling waves
- For $\beta = \pi$ then Z_0 is same as if $\beta = 0$

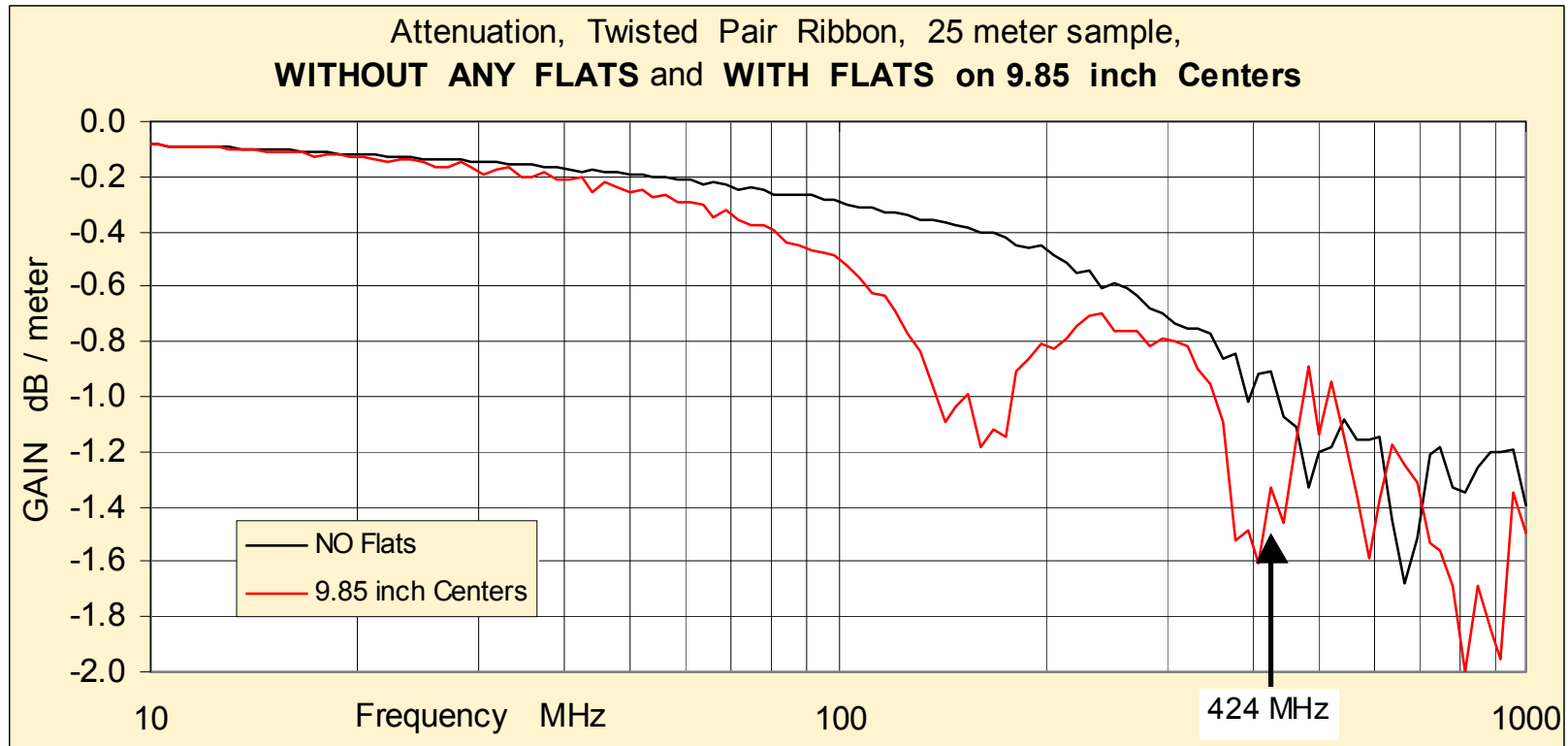
$$\cosh \alpha d = \left| \cos \frac{\theta}{2} - \frac{b}{2} \sin \theta \right| \geq 1$$

Bloch impedance

- Defined as the characteristic impedance of waves on the structure:
 - impedance is normally less than the impedance of the structure
 - can be as little as $0.3 Z_0$
- Impedance shift causes attenuation and reflection issues
- Can be calculated from the ABCD matrix

$$Z_{\pm} = \frac{\pm B Z_0}{\sqrt{A^2 - 1}}$$

Attenuation and impedance example



Terminated periodic structures

- Are similar to SCSI backplanes
- Have a propagation velocity much slower than light
 - Velocity can be as little as $0.4 c$
- Have reflected waves
 - Γ is a function of the Bloch impedance
 - Bloch impedance can be as little as $0.3 Z_0$
- For no reflections, Bloch impedance must equal the feeding transmission line impedance
 - Achieved through matching sections
 - Achieved by raising backplane impedance

Conclusions

- Periodic structure analysis applies to SCSI elements
 - Backplanes with connectors, w/wo drives
 - Ribbon cable with connectors, w/wo drives
 - Flat sections of twisted-flat cable
- Period structures have the following properties
 - Decreased impedance
 - Comb filter characteristics
 - Decreased propagation velocity
- SCSI backplane and cable performance can be analyzed mathematically through use of periodic structures.

References

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- R. E. Collin, *Foundations for Microwave Engineering*, McGraw-Hill, N.Y., 1966