

# Effects of Periodic Structures on Transmission Lines

---

# Why examine periodic structures?

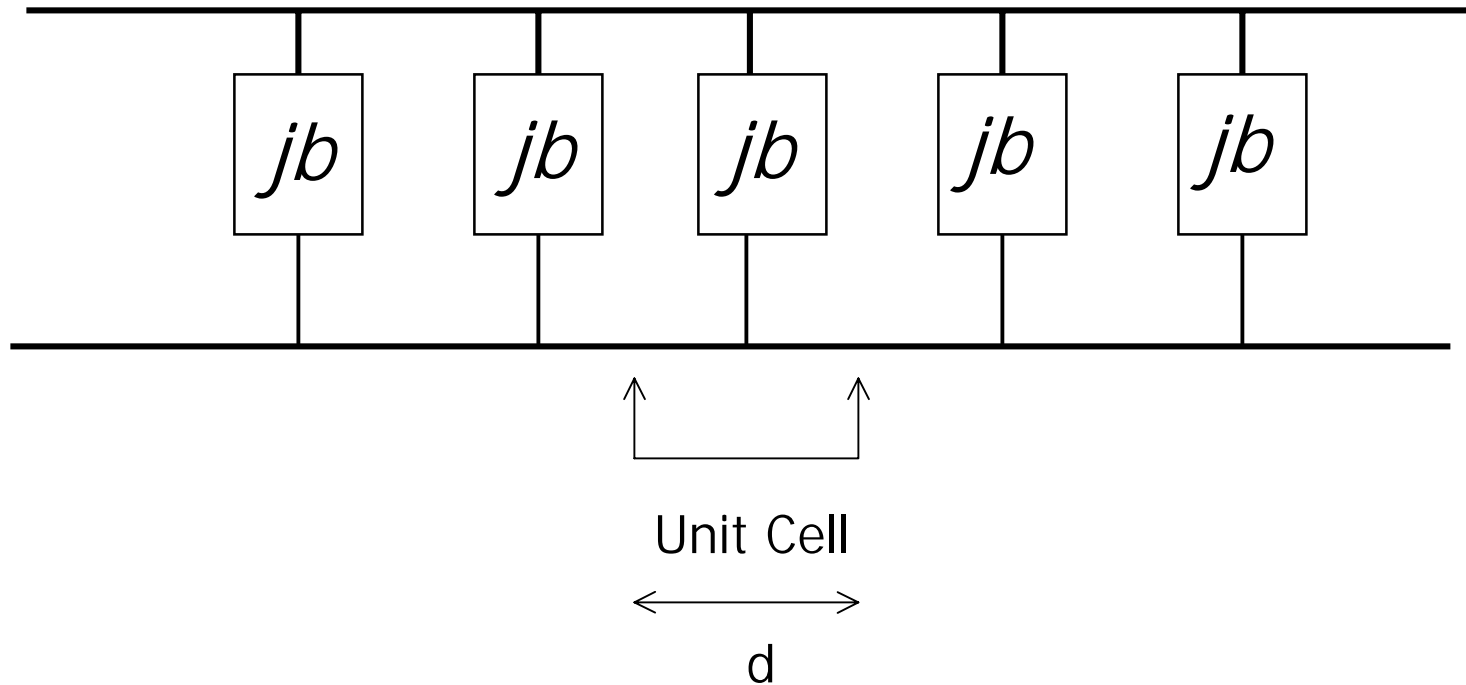
- Periodic structures - a definition
  - Transmission lines loaded at periodic intervals with identical reactive elements
- The following are examples of periodic structures:
  - A backplane loaded with SCSI devices
  - A ribbon cable loaded with SCSI devices
  - A bare twisted - flat ribbon cable with or without connectors
  - Periodic stubs on a microstrip line.
- The theory of periodic structures:
  - Provides insight into the behavior of backplanes and cables
  - Explains this behavior quantitatively

# Periodic structures

---

- Are characterized by a uniform distribution of reactive elements
- Have pass-band and stop-band properties
- Have slow wave properties
- Can exhibit significant impedance shifts
- Can be described mathematically through the use of voltage and current transfer functions

# An equivalent circuit



## Some definitions

---

- Propagation constant or wavenumber  $k = \sqrt{\mu\varepsilon}$
- Complex propagation constant  $\gamma = j\omega\sqrt{\mu\varepsilon}$
- Phase constant  $\beta = \text{Im}(\gamma)$
- Attenuation constant  $\alpha = \text{Re}(\gamma)$

# Characteristics

---

- Unloaded line:
  - Has a propagation constant of  $k$
  - Has a characteristic impedance of  $Z_0$
- Structure consists of a number of unit cells
  - Consist of a length  $d$  of transmission line
  - Have a shunt susceptance  $b$  across the midpoint of the cell
  - The susceptance is normalized to  $Z_0$
  - Can be represented by a cascade of identical 2-port networks

# Network representation

- Structure is a cascade of identical 2 port networks
- Each cell is represented by an identical ABCD matrix
- The ABCD matrix represents
  - Section of transmission line of length  $d/2$
  - A shunt susceptance of  $b$
  - Another section of transmission line of length  $d/2$
- Voltage and currents on either side of each cell are represented by

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}$$

# ABCD Matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \left( \cos \frac{\theta}{2} - \frac{b}{2} \sin \theta \right) & j \left( \sin \theta + \frac{b}{2} \cos \theta - \frac{b}{2} \right) \\ j \left( \sin \theta + \frac{b}{2} \cos \theta + \frac{b}{2} \right) & \left( \cos \theta - \frac{b}{2} \sin \theta \right) \end{bmatrix}$$

- $\theta = k \bullet d$  where  $k$  is the propagation constant of the unloaded line



# Current and voltage phase

- The voltage and current phase only differ by the propagation factor  $e^{-\gamma z}$

$$V_{n+1} = V_n e^{-\gamma z}$$

$$I_{n+1} = I_n e^{-\gamma z}$$

- Resulting in the following representation

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} V_{n+1} e^{\gamma d} \\ I_{n+1} e^{\gamma d} \end{bmatrix}$$

# Solution of matrix

- The former equation reduces to the following

$$AD + e^{2\gamma d} - (A + D)e^{\gamma d} - BC = 0$$

- Since  $\gamma = \alpha + j\beta$

- We have the following

$$\cosh \gamma d = \cosh \alpha d \cos \beta d + j \sinh \alpha d \sin \beta d = \cos \theta - \frac{b}{2} \sin \theta$$

- And since the right-hand side of the equation must be purely real, either
  - $\alpha = 0$ , and  $\beta \neq 0$
  - or
  - $\beta = 0$  or  $\pi$ , and  $\alpha \neq 0$

## Solution case $\alpha = 0$ , and $\beta \neq 0$

---

- Corresponds to non-attenuating propagating wave
- Defines a passband
- Solved for  $\beta$  if magnitude of right-hand side is less than unity
- Infinite number of values of  $\beta$

$$\cos \beta d = \cos \phi - \frac{b}{2} \sin \theta$$

## Solution case $\alpha \neq 0, \beta = 0, \pi$

- Wave is attenuated
- Defines stopband of structure
- Causes reflections back to input
- Has only one solution for positively traveling waves
  - $\alpha > 0$  for positively traveling waves
  - $\alpha < 0$  for negatively traveling waves
- For  $\beta = \pi$  then  $Z_0$  is same as if  $\beta = 0$

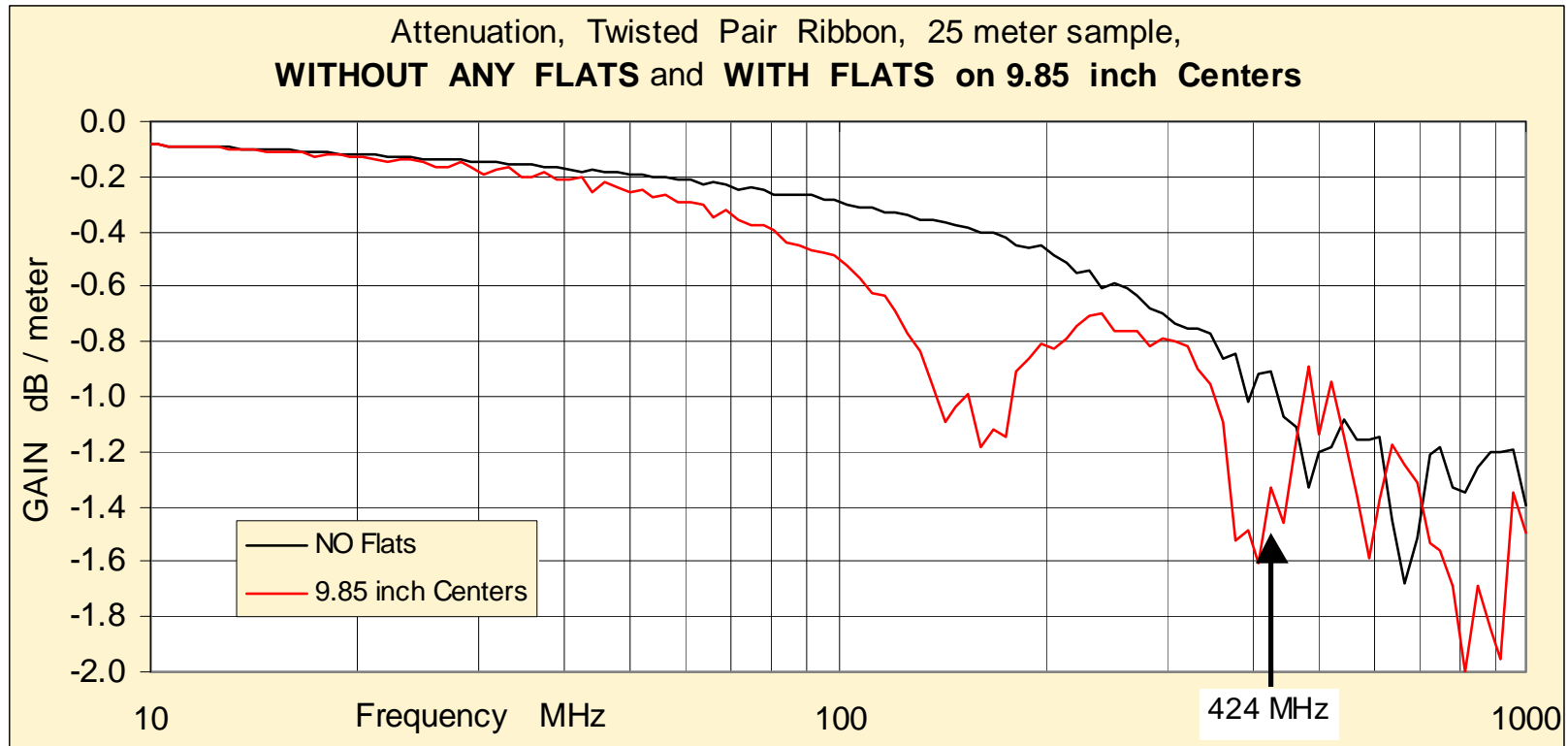
$$\cosh \alpha d = \left| \cos \frac{\theta}{2} - \frac{b}{2} \sin \theta \right| \geq 1$$

# Bloch impedance

- Defined as the characteristic impedance of waves on the structure:
  - impedance is normally less than the impedance of the structure
  - can be as little as  $0.3 Z_0$
- Impedance shift causes attenuation and reflection issues
- Can be calculated from the ABCD matrix

$$Z_{\pm} = \frac{\pm B Z_0}{\sqrt{A^2 - 1}}$$

# Attenuation and impedance example



# Terminated periodic structures

---

- Are similar to SCSI backplanes
- Have a propagation velocity much slower than light
  - Velocity can be as little as  $0.4 c$
- Have reflected waves
  - $\Gamma$  is a function of the Bloch impedance
  - Bloch impedance can be as little as  $0.3 Z_0$
- For no reflections, Bloch impedance must equal the feeding transmission line impedance
  - Achieved through matching sections
  - Achieved by raising backplane impedance

# Conclusions

---

- Periodic structure analysis applies to SCSI elements
  - Backplanes with connectors, w/wo drives
  - Ribbon cable with connectors, w/wo drives
  - Flat sections of twisted-flat cable
- Period structures have the following properties
  - Decreased impedance
  - Comb filter characteristics
  - Decreased propagation velocity
- SCSI backplane and cable performance can be analyzed mathematically through use of periodic structures.



# References

---

- D. M. Pozar, *Microwave Engineering*, Addison-Wesley, N.Y., 1990
- Ramo, Whinnery, & Van Duzer, *Fields and Waves in Communication Electronics*, 2nd Edition, Wiley, N.Y., 1967
- R. E. Collin, *Foundations for Microwave Engineering*, McGraw-Hill, N.Y., 1966